

Exercise 46

(a) Use a graph of

$$f(x) = \sqrt{3x^2 + 8x + 6} - \sqrt{3x^2 + 3x + 1}$$

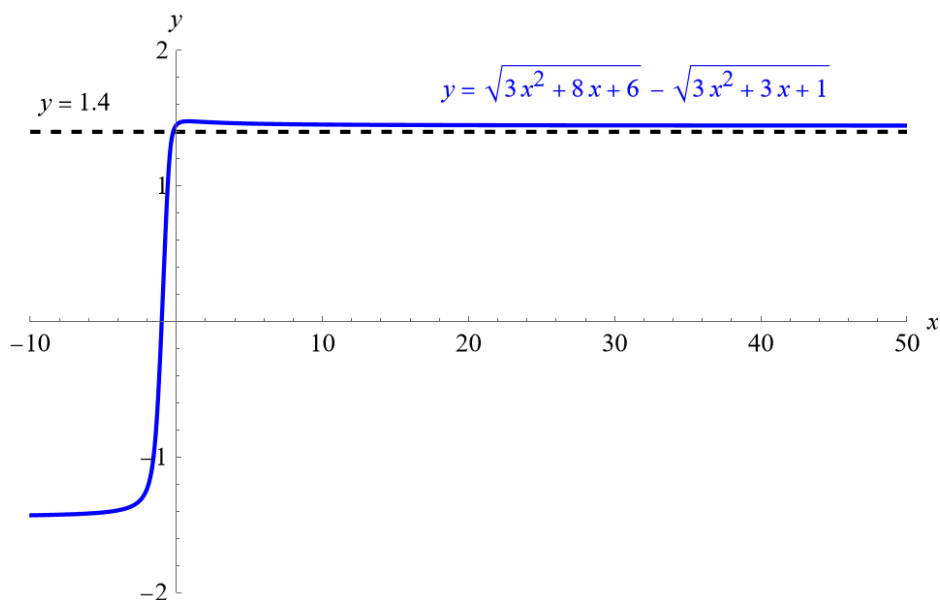
to estimate the value of $\lim_{x \rightarrow \infty} f(x)$ to one decimal place.

(b) Use a table of values of $f(x)$ to estimate the limit to four decimal places.

(c) Find the exact value of the limit.

Solution**Part (a)**

Graph the function versus x .



Based on the graph, the limit of the function as $x \rightarrow \infty$ is about 1.4.

$$\lim_{x \rightarrow \infty} f(x) \approx 1.4$$

Part (b)

Make a table with large values of x to see what happens as $x \rightarrow \infty$.

| x | $f(x)$ |
|-------|---------|
| 1 | 1.47735 |
| 10 | 1.45348 |
| 100 | 1.44456 |
| 1 000 | 1.4435 |

Part (c)

Compute the limit of $f(x)$ as $x \rightarrow \infty$.

$$\begin{aligned}
 \lim_{x \rightarrow \infty} \left(\sqrt{3x^2 + 8x + 6} - \sqrt{3x^2 + 3x + 1} \right) &= \lim_{x \rightarrow \infty} \left(\sqrt{3x^2 + 8x + 6} - \sqrt{3x^2 + 3x + 1} \right) \times \frac{\sqrt{3x^2 + 8x + 6} + \sqrt{3x^2 + 3x + 1}}{\sqrt{3x^2 + 8x + 6} + \sqrt{3x^2 + 3x + 1}} \\
 &= \lim_{x \rightarrow \infty} \frac{(3x^2 + 8x + 6) - (3x^2 + 3x + 1)}{\sqrt{3x^2 + 8x + 6} + \sqrt{3x^2 + 3x + 1}} \\
 &= \lim_{x \rightarrow \infty} \frac{5x + 5}{\sqrt{x^2 \left(3 + \frac{8}{x} + \frac{6}{x^2} \right)} + \sqrt{x^2 \left(3 + \frac{3}{x} + \frac{1}{x^2} \right)}} \\
 &= \lim_{x \rightarrow \infty} \frac{5x + 5}{x \sqrt{3 + \frac{8}{x} + \frac{6}{x^2}} + x \sqrt{3 + \frac{3}{x} + \frac{1}{x^2}}} \\
 &= \lim_{x \rightarrow \infty} \frac{5x + 5}{x \left(\sqrt{3 + \frac{8}{x} + \frac{6}{x^2}} + \sqrt{3 + \frac{3}{x} + \frac{1}{x^2}} \right)} \\
 &= \lim_{x \rightarrow \infty} \frac{5 + \frac{5}{x}}{\sqrt{3 + \frac{8}{x} + \frac{6}{x^2}} + \sqrt{3 + \frac{3}{x} + \frac{1}{x^2}}} \\
 &= \frac{5 + 0}{\sqrt{3 + 0 + 0} + \sqrt{3 + 0 + 0}} \\
 &= \frac{5}{2\sqrt{3}} \\
 &\approx 1.44338
 \end{aligned}$$